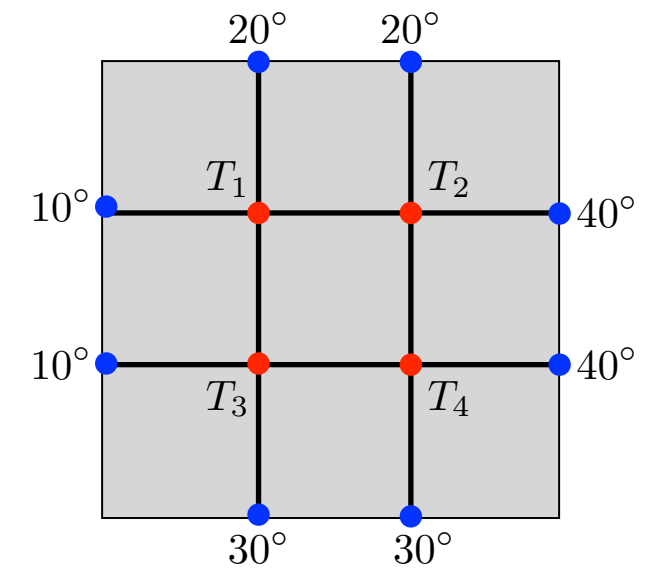


Equations Linéaires, Exemple : Transferts thermiques

Etude des Transferts Thermiques

- Trouver la température aux points rouges en sachant que pour chaque noeud, la température est la moyenne des températures aux quatre noeuds voisins

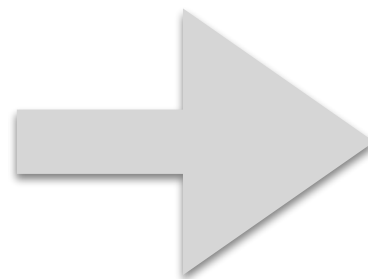


$$T_1 = \frac{1}{4}(10 + 20 + T_2 + T_3)$$

$$T_2 = \frac{1}{4}(T_1 + 20 + 40 + T_4)$$

$$T_3 = \frac{1}{4}(10 + T_1 + T_4 + 30)$$

$$T_4 = \frac{1}{4}(T_3 + T_2 + 40 + 30)$$



$$4T_1 - T_2 - T_3 = 30$$

$$-T_1 + 4T_2 - T_4 = 60$$

$$-T_1 + 4T_3 - T_4 = 40$$

$$-T_2 - T_3 + 4T_4 = 70$$

Resolution du système

$$\begin{aligned}
 4T_1 - T_2 - T_3 &= 30 \\
 -T_1 + 4T_2 - T_4 &= 60 \\
 -T_1 + 4T_3 - T_4 &= 40 \\
 -T_2 - T_3 + 4T_4 &= 70
 \end{aligned}
 \quad \Rightarrow \quad
 \left(\begin{array}{cccc|c}
 4 & -1 & -1 & 0 & 30 \\
 -1 & 4 & 0 & -1 & 60 \\
 -1 & 0 & 4 & -1 & 40 \\
 0 & -1 & -1 & 4 & 70
 \end{array} \right)
 \Rightarrow
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 4 & -1 & -1 & 0 & 30 \\
 -1 & 0 & 4 & -1 & 40 \\
 0 & -1 & -1 & 4 & 70
 \end{array} \right)
 \Rightarrow
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & 15 & -1 & -4 & 270 \\
 0 & -4 & 4 & 0 & -20 \\
 0 & -1 & -1 & 4 & 70
 \end{array} \right)$$

$$\left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & 0 & 1 & -2 & -75/2 \\
 0 & 0 & 0 & 24 & 720
 \end{array} \right)
 \xleftarrow{\times \frac{1}{24}}
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & 0 & 1 & -2 & -75/2 \\
 0 & 0 & -16 & 56 & 1320
 \end{array} \right)
 \xleftarrow{\times \frac{1}{8}}
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & 0 & 8 & -16 & -300 \\
 0 & 0 & -16 & 56 & 1320
 \end{array} \right)
 \xleftarrow{\times \frac{1}{8}}
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & -4 & 4 & 0 & -20
 \end{array} \right)$$

$$\left(\begin{array}{cccc|c}
 -1 & 4 & 0 & -1 & 60 \\
 0 & -1 & -1 & 4 & 70 \\
 0 & 0 & 1 & -2 & -75/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)
 \Rightarrow
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & 0 & 90 \\
 0 & -1 & -1 & 0 & -50 \\
 0 & 0 & 1 & 0 & 45/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)
 \Rightarrow
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & 0 & 90 \\
 0 & -1 & 0 & 0 & -55/2 \\
 0 & 0 & 1 & 0 & 45/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)
 \xrightarrow{\times (-1)}
 \left(\begin{array}{cccc|c}
 -1 & 4 & 0 & 0 & 90 \\
 0 & 1 & 0 & 0 & 55/2 \\
 0 & 0 & 1 & 0 & 45/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)$$

$$\left(\begin{array}{cccc|c}
 -1 & 0 & 0 & 0 & -20 \\
 0 & 1 & 0 & 0 & 55/2 \\
 0 & 0 & 1 & 0 & 45/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)
 \xrightarrow{\times (-1)}
 \left(\begin{array}{cccc|c}
 1 & 0 & 0 & 0 & 20 \\
 0 & 1 & 0 & 0 & 55/2 \\
 0 & 0 & 1 & 0 & 45/2 \\
 0 & 0 & 0 & 1 & 30
 \end{array} \right)
 \Rightarrow
 T_1 = 20, T_2 = 27.5, T_3 = 22.5, T_4 = 30$$